(For publication in ARS Journal)

N65-83279

Code none

nasa TMX 56247

ON A METHOD FOR ACHIEVING SATELLITE RENDEZVOUS

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Many proposed space missions will require achieving rendezvous of two bodies in an orbit about a planet. For example, an interplanetary transport vehicle may wish to transfer passengers to an orbiting space station. It is generally most efficient to control the spaceship during its approach to the final orbit altitude in such fashion that rendezvous is accomplished immediately upon entering orbit. (Methods of transferring between dissimilar orbits with simultaneous rendezvous are discussed in references 1 and 2.) However as a result of guidance errors, unaccounted-for perturbative forces, etc., it is unlikely that perfect rendezvous will be achieved in this fashion. The situation may then be envisioned where the spaceship has established itself in the desired orbit (assumed to be circular), but is separated from the target body by some angle.

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Several procedures are conceivable for correcting ~~this~~ angular error. With high-thrust engines it is possible for the spaceship to be transferred by means of an impulsive thrust to an elliptical trajectory that returns to the circular orbit at the proper instant for rendezvous (ref. 2). With low-thrust engines (such as ion or plasma rockets), however, thrust must be applied continuously during an appreciable time interval in order to change the velocity or path of

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the spaceship. One expedient is to continuously direct the thrust in a more or less circumferential direction in order to spiral some distance away from the circle. The distance is chosen such that, upon spiraling back to the circle, rendezvous is achieved.

In this note an alternative method is considered in which the spaceship remains in the circular orbit while it accomplishes rendezvous with the target. The velocity increments and times required for this method are compared with those for the techniques mentioned above.

Nomenclature

F	engine thrust
m	spaceship mass
r	distance from center of earth
t	elapsed time
v_c	circular orbit velocity
W	weight of spaceship at sea level
α	angle between thrust and velocity vectors
δ	angular separation between the target and spaceship
θ	angular distance traveled
μ	gravitational force constant for earth

Analysis

The initial thrust applied to the spaceship is directed circumferentially. As the velocity of the spaceship changes, the thrust vector must be rotated so that its radial component compensates for the resulting imbalance between the centrifugal and gravitational forces in order to maintain the spaceship in the circular orbit (fig. 1).

If it is assumed that the thrust-to-weight ratio of the spaceship is constant during the rendezvous maneuver, the differential equations of motion may be solved explicitly for this method of flight, yielding the relations,

$$\alpha = \alpha_0 - 2\theta \tag{1}$$

$$\Delta t = \frac{r^{3/2}}{\sqrt{\mu}} \int_0^\theta \frac{d\theta}{\sqrt{1 + \frac{F}{W} \sin 2\theta}} \tag{2}$$

Initially α is 0° if the target is ahead of the spaceship (180° if the target is behind). Equation (1) shows that the thrust angle varies linearly with the distance traveled. Note that upon completion of 45° of spaceship travel, the full thrust vector is oriented radially in order to hold the spaceship in the circular orbit and the vehicle can no longer be accelerated.

Equation (2) can be transformed into an elliptic integral of the first kind and evaluated with the use of tables. In the period of time Δt, the spaceship and the target travel through the angular distances θ and $v_c \Delta t/r$ respectively. The difference between these displacements is the amount by which the initial angular separation, δ_0, is reduced. The thrust vector at time Δt is at an angle α_1 with the circumferential direction. To restore the spaceship to the circular orbit velocity, the thrust vector is reversed (mirror wise) to an angle of $180-\alpha_1$ degrees and then rotated back to a circumferential position, ending up, however, opposite to the initial thrust direction. The further reduction in angular separation during this deceleration phase is equal to the reduction accomplished during the initial acceleration period; also each phase of the maneuver takes time Δt. The maximum amount, δ_M, by which the initial

velocity increments and times shown for the impulsive-thrust and spiral-trajectory methods are for separation angles in which the target leads the spaceship, while those for the constant-radius trajectory are independent of whether the target leads or lags the spaceship.) The constant-radius technique is definitely inferior to the other methods in terms of both the time and the velocity increment required for rendezvous. For a separation of 0.25° (which is only about 20 miles at an altitude of 500 miles) a ΔV of nearly 300 feet per second is necessary for a thrust-to-weight ratio of 10^{-4}. With sufficiently high values of specific impulse the value of ΔV needed for rendezvous may not correspond to a very large fuel consumption; however the spaceship crew would not care to take the indicated time of 24 hours to complete the last 20 miles of their journey.

The constant-radius method of achieving final rendezvous does not, therefore, appear desirable unless available thrust-to-weight ratios are much higher than 10^{-4}, and then only if further study indicates some advantage (e.g., guidance) over the spiral-trajectory method.

References

1. Paiewonsky, B. H., "Transfer Between Vehicles in Circular Orbits" W.A.D.C. Report TN 57-267; August 1957.

2. Brunk, W. E., and Flaherty, R. J., "Methods and Velocity Requirements for the Rendezvous of Satellites in Circumplanetary Orbits" Unpublished NASA data.

angular separation, δ_0, may be reduced during the acceleration period plus a similar deceleration period is shown in figure 2.

If the initial angular separation is greater than the value shown in figure 2, the thrust may be held in the intermediate radial position and the spaceship allowed to "coast" at constant velocity for a long enough period of time to make up the difference between δ_0 and δ_M. The rate of closure during the coasting phase is given by equation (3) and plotted in figure 3.

$$\dot{\delta} = \frac{180}{\pi}\frac{v_c}{r}\left(\sqrt{1 + \frac{F}{W}} - 1\right) \qquad (3)$$

As a measure of the fuel expenditure necessary to carry out a rocket mission it is convenient to consider the characteristic velocity increment that would be achieved by the vehicle in linear gravitation-free flight. This velocity increment may be calculated for the constant-radius rendezvous maneuver from equation (4)

$$\Delta V = \int_0^{t_{total}} \frac{F}{\underset{m}{\cancel{M}}}\,dt = \left(\frac{F}{\underset{m}{\cancel{M}}}\right)t_{total} \qquad (4)$$

where $t_{total} = 2\Delta t + t_{coast}$.

The times and velocity increments required to correct for various angular separations are shown in figures 4 and 5. Note that both of these quantities are quite large for all but the smallest values of δ_0 in the range of thrust-to-weight ratios anticipated for continuous-thrust propulsion systems.

Discussion

A comparison among the various methods described for attaining rendezvous is made in figure 6 for several separation angles. (The

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Figure Legends

1. Forces acting on spaceship.
2. Maximum possible decrease in angular separation during acceleration and deceleration phases as a function of thrust-to-weight ratio.
3. Rate of closure during coasting as a function of thrust-to-weight ratio for two orbit altitudes.
4. Rendezvous time as a function of separation angle for several values of F/W. Altitude = 500 miles.
5. Velocity increment required for rendezvous as a function of separation angle for several values of F/W. Altitude = 500 miles.
6. Velocity increments required for rendezvous as a function of rendezvous time. Altitude = 500 miles.

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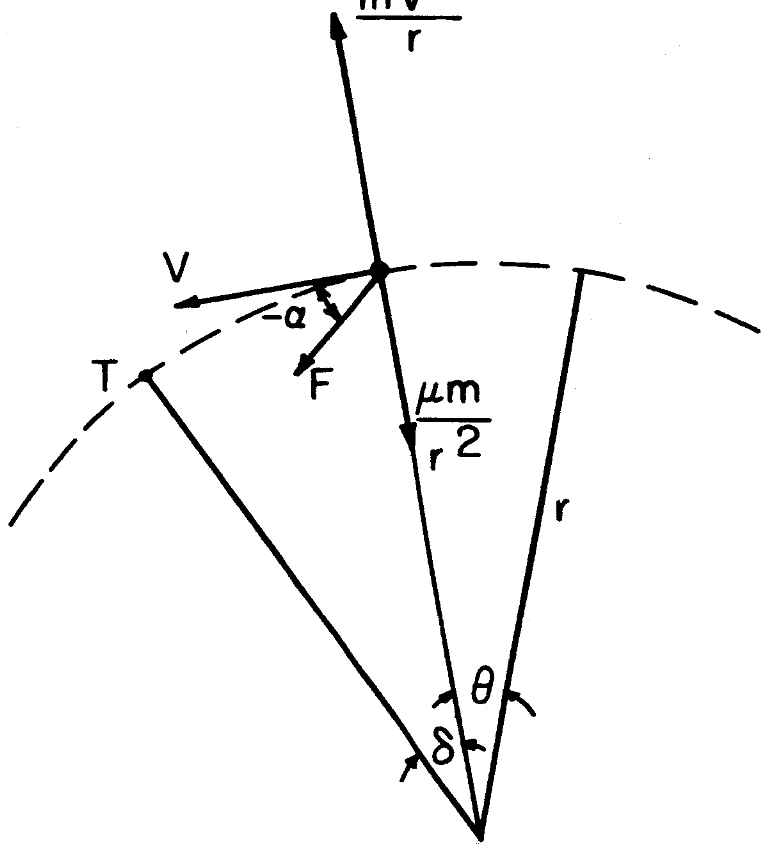

Fig. 1. - Forces acting on spaceship.

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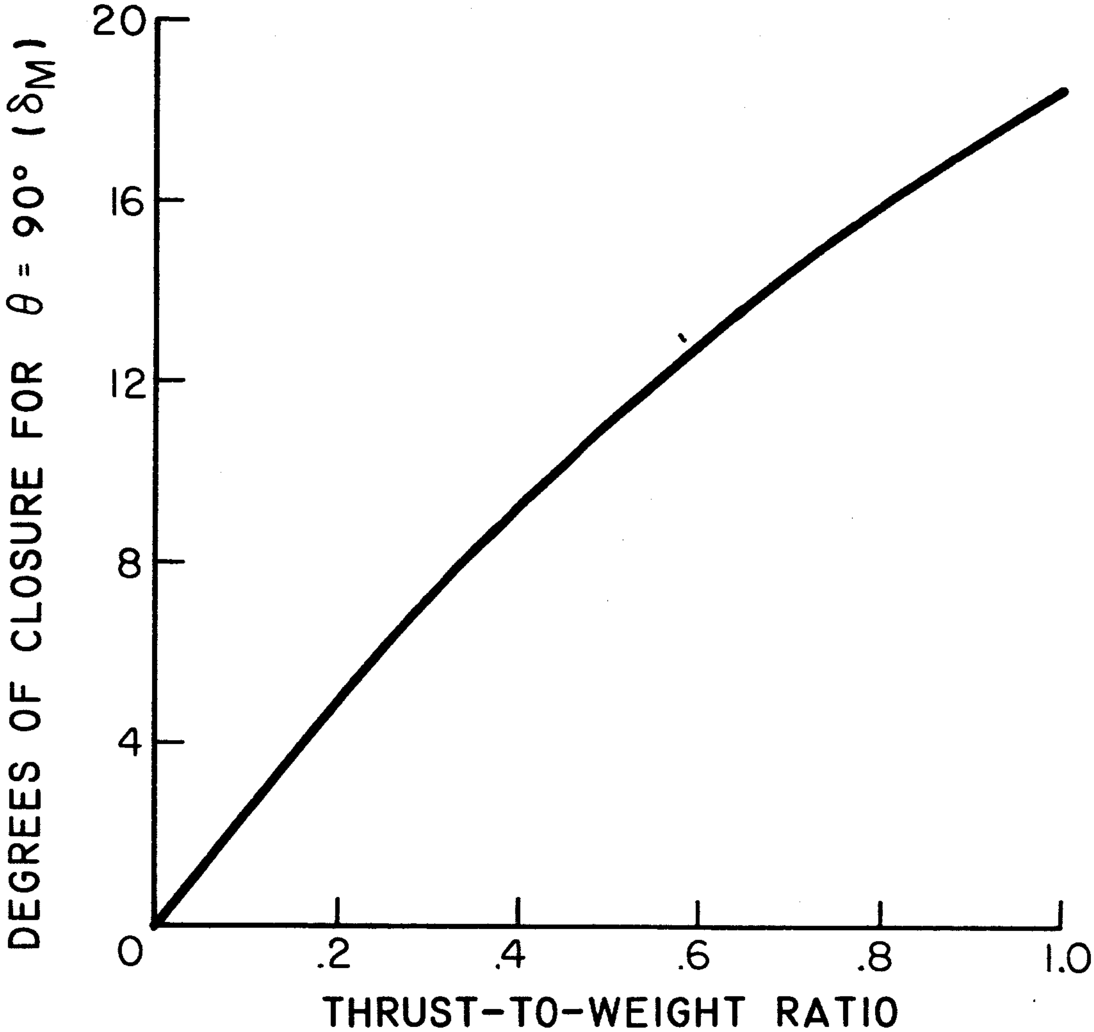

Fig. 2. - Maximum possible decrease in angular separation during acceleration and deceleration phases as a function of thrust-to-weight ratio.

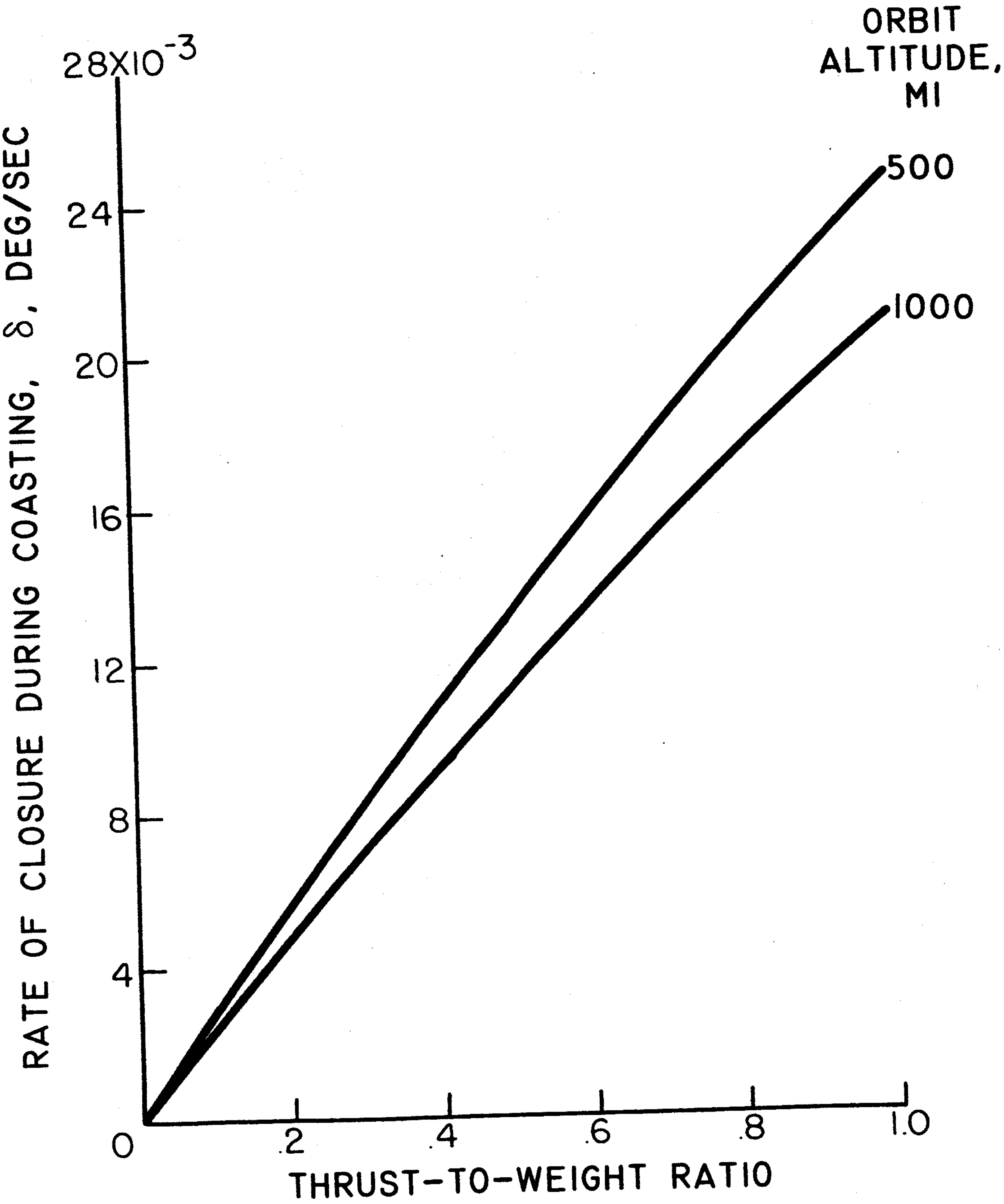

Fig. 3. - Rate of closure during coasting as a function of thrust-to-weight ratio for two orbit altitudes.

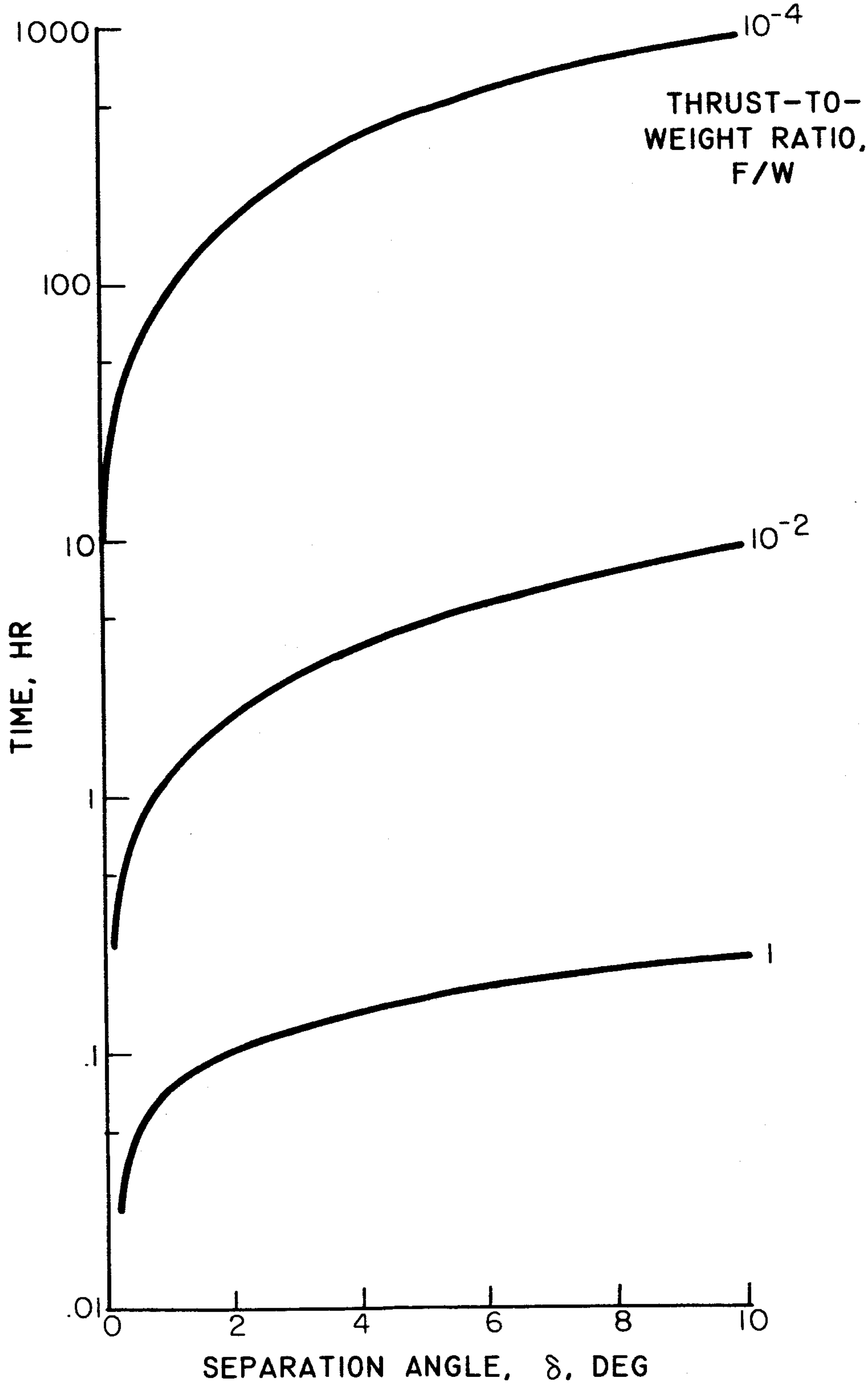

Fig. 4. - Rendezvous time as a function of separation angle for several values of F/W. Altitude = 500 miles.

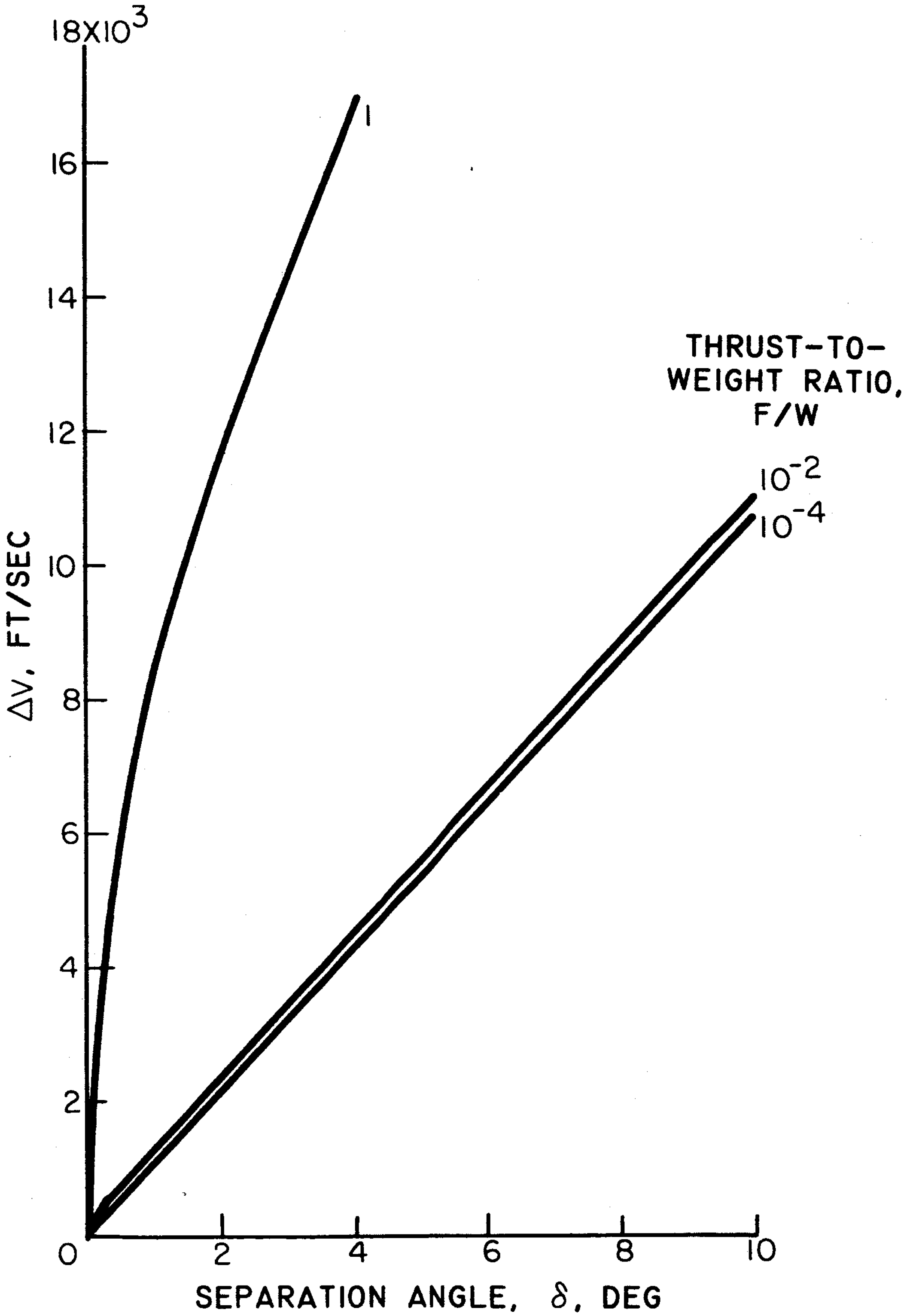

Fig. 5. - Velocity increment required for rendezvous as a function of separation angle for several values of F/W. Altitude = 500 miles.

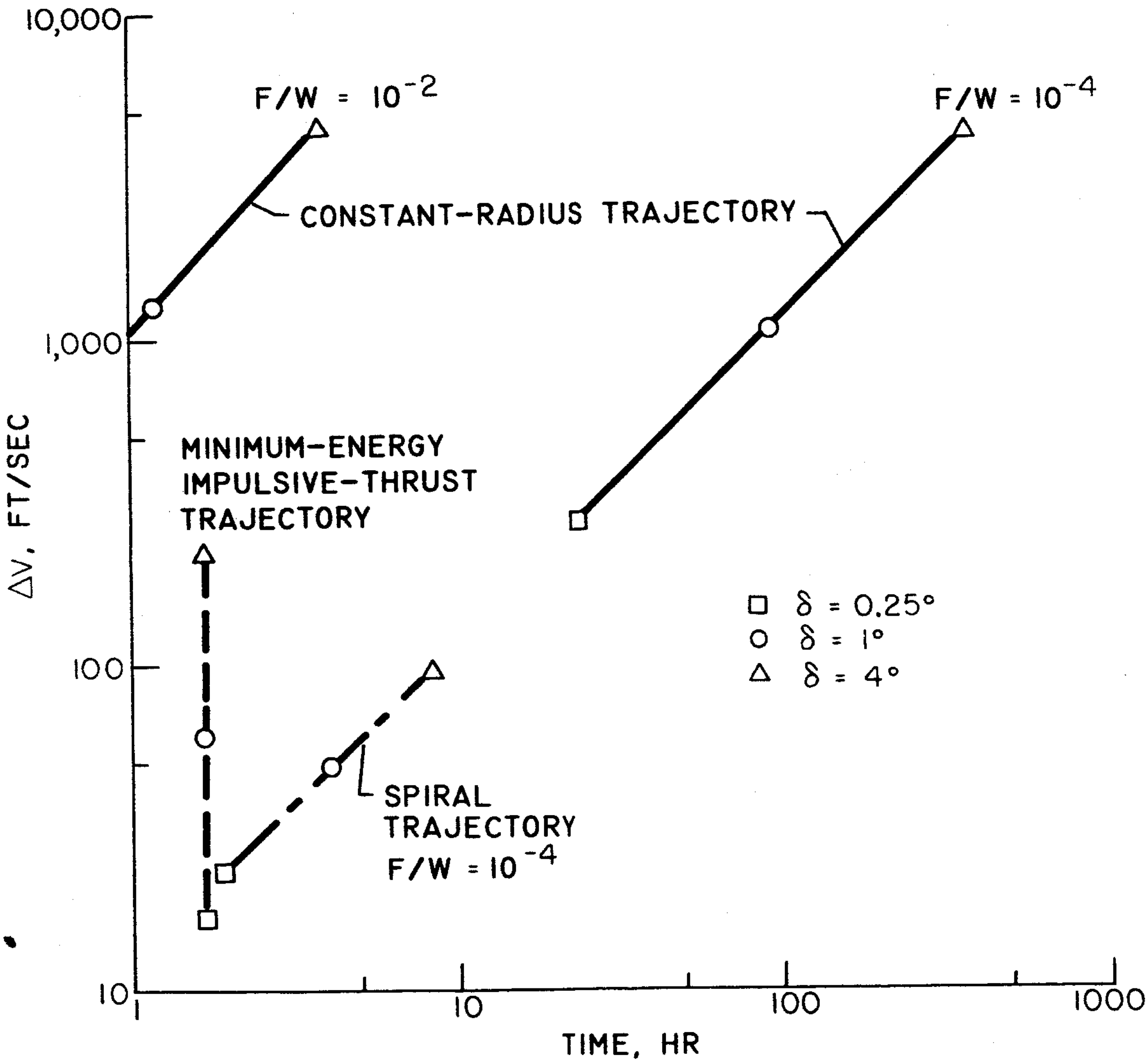

Fig. 6. - Velocity increments required for rendezvous as a function of rendezvous time. Altitude = 500 miles.